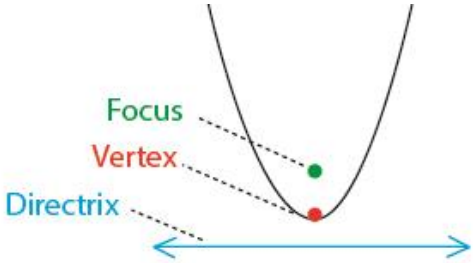
Parabolas

A parabola is the set of all points that are eqidistant from a given point (focus) and a given line (directrix).

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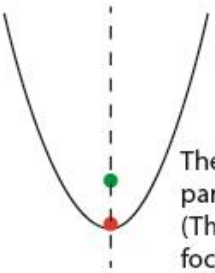
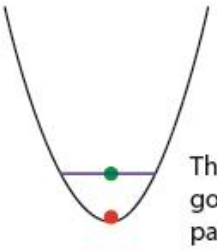
Other important aspects associated with parabolas:



The Axis of Symmetry divides the parabola into two equal sides. (The A of S goes through the focus AND the vertex)



The Latus Rectum is a chord that goes through the focus and is parallel to the directrix.

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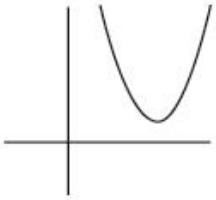
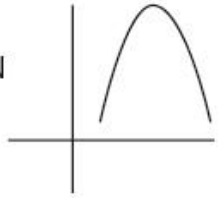
They can be modeled by the equation : $y = a(x - h)^2 + k$

if a > 0,
Graph opens UP



if a < 0,
Graph opens DOWN



Vertex: (h, k)

Axis of symmetry: $x = h$

Length of latus rectum = $\left|\frac{1}{a}\right|$

Focus: $\left(h, k + \frac{1}{4a}\right)$

Directrix: $y = k - \frac{1}{4a}$

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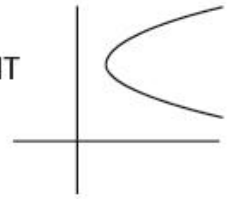
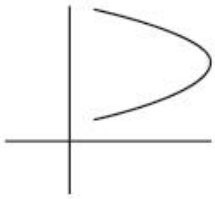
Or they can be modeled by the equation : $x = a(y - k)^2 + h$

if a > 0,
Graph opens RIGHT



if a < 0,
Graph opens LEFT



Vertex: (h, k)

Axis of symmetry: $y = k$

Length of latus rectum = $\left|\frac{1}{a}\right|$

Focus: $\left(h + \frac{1}{4a}, k\right)$

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Ex: Find the vertex and focus of the parabola: $y = 3(x - 6)^2 + 4$

a = 3, h = 6, k = 4 → Vertex = (6, 4) Focus = $\left(6, 4 + \frac{1}{4(3)}\right) = \left(6, 4 + \frac{1}{12}\right) = \left(6, \frac{49}{12}\right)$

Ex: Find the equation of the directrix and the length of the latus rectum for the parabola $x = \frac{1}{2}y^2 - 2$

$a = \frac{1}{2}, h = -2, k = 0$ → directrix = $y = -2 - \frac{1}{4 \cdot \frac{1}{2}} = -2 - \frac{1}{2} = -\frac{5}{2}$, latus rectum = $\left|\frac{1}{1/2}\right| = 2$